Feynman Diagrams

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Feynman diagrams are a pictorial way of describing integrals predicting possible outcomes of interactions of subatomic particles in the context of quantum field physics. As the number of variables that are being integrated out increases, the integrals become more and more difficult to compute. But in the cases when the integrals can be computed, the accuracy of their prediction is amazing. Many of these diagrams corresponding to real-world scenarios result in integrals that are divergent in mathematical sense. Physicists have a collection of competing techniques called "renormalization" of Feynman integrals which "cancel out the infinities" coming from different parts of the diagrams. After renormalization, calculations using Feynman diagrams match experimental results with very high accuracy. However, these renormalization techniques appear very suspicious to mathematicians and attract criticism from physicists as well. For example, do you get the same result if you apply a different technique? If the results are different, how do you choose the "right" technique? Or, if the results are the same, what is the reason for that? Most of these questions will be resolved if one finds an intrinsic mathematical meaning of Feynman diagrams, most likely as projectors onto irreducible components of certain representations. A number of mathematicians already work on this problem of finding a mathematical interpretation of Feynman diagrams, mostly in the setting of algebraic geometry. Recently published book "Feynman Motives" by M. Marcolli, [1] contains a summary of these algebraic-geometric developments as well as a comprehensive list of references. However, there is a strong evidence that at least some Feynman diagrams should have a representation-theoretic interpretation. The answer might be as simple and elegant as projectors onto irreducible components of certain representations of $U(2,2)$.

References

1. M. Marcolli, *Feynman Motives,* World Scientific Publ. Co., (2010)