Abstract for the special program *Eigenvarieties*

At present we are witnessing an important, and natural, expansion of the scope of the classical Langlands program. This new mathematical development makes use of the rich structure of congruences between Fourier coefficients of modular forms, and more generally of automorphic representations, to tie together infinitely many otherwise disparate automorphic representations into finite-dimensional parameter spaces. By one count, there seems to be six independent, essentially simultaneous constructions currently underway, of parametrized (p-adic) spaces of automorphic forms attached to algebraic groups, and their concomitant Galois representations. These parameter spaces are called "eigenvarieties," or "Hecke varieties," and are being constructed by different people, in different but sometimes overlapping contexts: for unitary groups of higher rank, for symplectic groups of high rank, for general linear groups over number fields. Eigenvarieties are a unifying force for classical and modern aspects of number theory, algebraic geometry, analytic geometry (p-adic, mainly) and the theory of group representations (both automorphic representations and Galois representations). Some of this work has already been used in important applications. The *Eigenvarieties* program at Harvard University during the Spring semester 2006 is intended to bring together many of the people working on these constructions to provide intensive graduate courses on this material and satellite seminars.

The classical work of Ramanujan, that dealt with the arithmetic properties of the Fourier coefficients of modular forms, unearthed striking congruences that contain important number theoretic information. These congruences suggest that a mysterious coherence underlies a large assortment of basic arithmetic phenomena such as *the number of ways you can separate a collection of N objects into subcollections*, or given a lattice in some Euclidean space, *the number of lattice points closest to a given point*, or *the number of solutions of a system of polynomial equations modulo a prime number*. An extraordinary web of congruences acts as a virtual glue that binds such problems together. In the intervening years, the search for *congruences* that have arithmetic applications, that unify representation theory, and the theory of modular forms, has guided much number-theoretic work. This search has been directly involved in many of the important advances in number theory in the past few decades. For example, it played its role in the dramatic proof of modularity of elliptic curves over the rational numbers, a few years ago. One is now on the verge of a significant expansion of this enterprise. The hope is that the *Eigenvarieties* program at Harvard University during the Spring semester 2006 program will provide a milieu where further progress can be made, where a coherent account of the current state of knowledge will be established, and where graduate students, and also post-docs and other interested mathematicians, can gain mastery of these new developments.