Mardi 18/04/06

Aspects of the p-adic local Langlands programme

Aspect III : Drinfel'd spaces

Intro. I would like to explain here a third aspect of p-adic Langlands, namely the link with Drinfel'd spaces (for $GL_2(\mathbb{Q}_p)$). This aspect has not proven to be very powerful so far (contrary to (φ,Γ)-modules), but it gives a new point of view on representations: first, it insists on locally analytic representations rather than Banach spaces and second, it is "geometric" (it uses the de Rham complex).

Let me remind to you the setting (for $GL_2(\mathbb{Q}_p)$).

Start with V = absolutely irred. repr. of $Gal(\overline{\mathbb{Q}}_p/\mathbb{Q}_p)$ on a 2-dim^l K-vector space. Then, using the (φ,Γ)-module associated to V, one can associate a topologically irred. representation of the Borel subgrp of $GL_2(\mathbb{Q}_p)$ $B(V)$. Assume V is de Rham (or p.s.st) with HT weights $(0,k-1)$ with $k\geqslant 2$, then one should have that $B(V)$ is isomorphic to a unitary completion of $Sym^{k-2}K^2\otimes_K \Pi(V)$, a completion "depending" on the Hodge filtration.

I would like to present here a method to construct, at least in some cases, completions of $Sym^{k-2}K^2\otimes_K \Pi(V)$ using locally analytic representations. The BIG drawback is that we don't even know the completion is non-zero.

The semi-stable case (non-crystalline).

Any abs. irr. V as above semi-stable with HT weights $(0,k-1)$ is such that, up to unramified twist:

$D = D_{st}(V) = Ke_1 \oplus Ke_0$ $\quad \begin{cases} \varphi(e_1) = p^{-\frac{k-2}{2}} e_1 \\ \varphi(e_0) = p^{-k/2} e_0 \end{cases}$ $\quad \begin{cases} N(e_1) = e_0 \\ N(e_0) = 0 \end{cases}$ $\quad \mathrm{Fil}^i D = \begin{cases} \text{all if } i \leq -(k-1) \\ K(e_1 + \mathcal{L} e_0) \text{ if } -(k-2) \leq i \leq 0 \\ 0 \text{ if } i \geq 1 \end{cases}$

$\underline{\underline{k > 2}}$ $\quad \mathcal{L} \in K$

We have $\pi(V) = \text{Steinberg} \otimes |\det|^{\frac{k-2}{2}}$. We know that, if Σ_0 denotes Drinfeld' rigid analytic half plane $/\mathbb{Q}_p$ (recall $\Sigma_0(\mathbb{C}_p) = \mathbb{C}_p \setminus \mathbb{Q}_p$), then $H^1_{dR}(\Sigma_0) \simeq \dfrac{\Omega^1(\Sigma_0)}{d\mathcal{O}(\Sigma_0)} \simeq \text{Steinberg}^{\vee}$ (algebraic dual).

So the idea is: try and use Σ_0 to build a representation where one has a parameter $\mathcal{L}$, related to the p-adic logarithm.

Recall that $\Omega^1(\Sigma_0) \xrightarrow{\sim} \{\text{Rigid analytic functions on } \Sigma_0\}$

$$f(z)\,dz \mapsto f(z)$$

with the action of $GL_2(\mathbb{Q}_p)$ given by $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot f = \dfrac{ad-bc}{(bz+d)^2} f\left(\dfrac{az+c}{bz+d}\right)$.

Define $\mathcal{O}(k) = \{\text{rigid analytic functions on } \Sigma_0 \text{ which are } K\text{-rational}\}$

with the action of $GL_2(\mathbb{Q}_p)$ given by $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot f = |ad-bc|^{-\frac{k-2}{2}} \dfrac{ad-bc}{(bz+d)^k} f\left(\dfrac{az+c}{bz+d}\right)$

[by K-rational, I mean which can be written $\sum_{n=0}^{+\infty} b_n z^n + \sum_i \sum_{n=1}^{\infty} \dfrac{b_{i,n}}{(z-z_i)^n}$ on an affinoïd (for some $z_i \in \mathbb{Q}_p$) with $b_n, b_{i,n} \in K$]. So essentially $\mathcal{O}(2) = \Omega^1(\Sigma_0)$. The space $\mathcal{O}(k)$ is naturally a Fréchet space, writing:

$$\mathcal{O}(k) = \varprojlim_{U \text{ affinoïd}} \mathcal{O}(k)_U \quad \text{where } \mathcal{O}(k)_U = \text{functions as in}$$

$(U = \mathbb{P}^1(\mathbb{C}_p) \setminus \text{disks around } z_i \text{ and } \infty) \subset \mathbb{C}_p \setminus \mathbb{Q}_p$

and the action of GL_2 is continuous

Let $\log_{\mathcal{L}} : \mathbb{C}_p \setminus \{0\} \to \mathbb{C}_p$ the unique branch of the p-adic logarithm such that $\log_{\mathcal{L}}(p) = \mathcal{L}$. For U an affinoïd as above, define: $(b_n, b_{i,n}, c_{i,n} \in K)$

$\mathcal{O}(k, \mathcal{L})_U$ = functions $U \to \mathbb{C}_p$ of the form $f(z) = \sum_{n=0}^{+\infty} b_n z^n + \sum_i \sum_{n=1}^{\infty} \dfrac{b_{i,n}}{(z-z_i)^n} + \sum_i \sum_{n=0}^{k-2} c_{i,n} z^n \log_{\mathcal{L}}(z-z_i)$

then $\mathcal{O}(k,\mathcal{L}) := \varprojlim_{U} \mathcal{O}(k,\mathcal{L})_U$ is again a Fréchet space and one can endow it with a continuous action of $GL_2(\mathbb{Q}_p)$ by: ③

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} f = |ad-bc|^{-\frac{k-2}{2}} \frac{(bz+d)^{k-2}}{(ad-bc)^{k-2}} f\left(\frac{az+c}{bz+d}\right).$$

<u>Thm</u>: (i) $\mathcal{O}(k,\mathcal{L})^\vee$ (continuous dual) is a locally analytic representation of $GL_2(\mathbb{Q}_p)$ that has 3 topological JH factors which are:

$$\underbrace{\underset{\substack{\uparrow \\ \text{unique irred. subobj.}}}{\mathrm{Sym}^{k-2}K^2 \otimes \mathrm{Steinberg} \otimes |\det|^{\frac{k-2}{2}}}, \left(\mathrm{Ind}_B^G a^{k-1} \otimes d^{-1}\right)^{an} \otimes |\det|^{\frac{k-2}{2}}}_{\mathcal{O}(k)^\vee}, \underset{\substack{\uparrow \\ \text{unique irred. quotient}}}{\mathrm{Sym}^{k-2}K^2 \otimes |\det|^{\frac{k-2}{2}}}$$

(ii) The universal unitary completion of $\left|\begin{matrix}\mathcal{O}(k)^\vee \\ \text{Irr. subobj.}\end{matrix}\right.$ is isomorphic to the Banach space of functions $f : \mathbb{Q}_p \to K$ that are of class $C^{\frac{k-2}{2}}$ (see transparency of Aspect II) such that $x^{k-2} f(\frac{1}{x})$ is of class $C^{\frac{k-2}{2}}$ modulo polyn. of degree $\leq k-2$ and with the action of $GL_2(\mathbb{Q}_p)$:

argument similar to previous talk.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} f = |ad-bc|^{\frac{k-2}{2}} (bx+d)^{k-2} f\left(\frac{ax+c}{bx+d}\right).$$

$f(x) = \sum_{n=0}^{+\infty} a_n \binom{x}{n}$, $n^{\frac{k-2}{2}}|a_n| \to 0$

(iii) The universal unitary completion of the 2 other JH is 0.

(iv) The universal unitary completion of $\mathcal{O}(k,\mathcal{L})^\vee$ is isomorphic to the quotient:

$$B(V) := \frac{\text{universal unitary completion of } \mathrm{Sym}^{k-2}K^2 \otimes \mathrm{Steinberg} \otimes |\det|^{\frac{k-2}{2}}}{\left(\begin{matrix}\text{closure of subspace of } f \text{ of the form } \sum_{i \in I} \lambda_i (z-a_i)^{n_i} \log_{\mathcal{L}}(z-a_i) \\ \text{where } \frac{k-2}{2} < n_i \leq k-2 \text{ and } \deg\left(\sum_{i\in I} \lambda_i (z-a_i)^{n_i}\right) < \frac{k-2}{2} \\ a_i \in \mathbb{Q}_p\end{matrix}\right)}$$

<u>universal unitary completion</u> (Emerton):

Let W be a locally convex topological K-vector space endowed with a continuous $GL_2(\mathbb{Q}_p)$-action. Consider the functor from $GL_2(\mathbb{Q}_p)$-unitary Banach spaces to sets : $B \mapsto \mathrm{Hom}_{GL_2(\mathbb{Q}_p)}(W, B)$ (continuous equiv. homom.).

If this functor is representable, we call the corresponding $GL_2(\mathbb{Q}_p)$-unitary Banach space $\hat{W}$ the universal unitary completion of W. Note that $W \to \hat{W}$ is necessarily dense (otherwise, take the closure of the image of W) but not injective in general ($\hat{W}$ can be 0).

There is a sufficient condition (due to Emerton) so that W admits a universal unitary completion, at least in the case W is a locally analytic representation. It is that $W^\vee$ (which is a reflexive Fréchet space) admits a continuous semi-norm:

$q : W^\vee \longrightarrow K$ such that the collection of semi-norms $(g.q \ , g \in GL_2(\mathbb{Q}_p))$ gives the topology of the Fréchet space $W^\vee$. In that case, it is easy to check that the universal completion is the dual of:

$$\{ v \in W^\vee \mid q(gv) \leq 1 \ \ \forall g \in GL_2(\mathbb{Q}_p) \} \otimes K \text{ (which doesn't depend on } q).$$

Ex: For instance, take $W^\vee = \mathcal{O}(h)$, $W^\vee = \mathcal{O}(h, \mathcal{Z})$, choose U affinoid such that $(gU)_g$ cover Σ_0, and consider the semi-norm:

$$q : \mathcal{O}(h, \mathcal{Z}) = \varprojlim_U \mathcal{O}(h, \mathcal{Z})_U \longrightarrow \mathcal{O}(h, \mathcal{Z})_U \xrightarrow[\mathcal{O}(h,\mathcal{Z})_U]{\text{norm on}} K$$

then the universal unitary completion of $\mathcal{O}(h, \mathcal{Z})^\vee = \left(\{ f \in \mathcal{O}(h, \mathcal{Z}), \ q(gf) \leq 1 \ \forall g \} \otimes K \right)^\vee$.

Rq: This is how the Banach space $B(V)$ was first found. Colmez proved it is $\neq 0$, admissible top. irreducible using (φ, Γ)-modules as in Lecture 2.

A supercuspidal case.

Now, we would like to extend the previous theorem to the supercuspi-dal case (this is a joint project with M. Strauch). So far, we have not been able to prove much, but I would like to explain the candidate we have in the most simple case, the case of weight 2 and of the first covering.

Consider V (abs.) irreducible, of Hodge-Tate weights $(0,1)$, which becomes crystalline over $\mathrm{Gal}(\overline{\mathbb{Q}_p}/F)$ where $F = \mathbb{Q}_{p^2}(\sqrt[p^2-1]{-p})$ $(= L')$. Then it is described by $D_{\mathrm{cris}}(V|_{\mathrm{Gal}(\overline{\mathbb{Q}_p}/F)}) \circlearrowleft \mathrm{Gal}(F/\mathbb{Q}_p)$ which is as follows (D. Savitt) up to twist:

$$\underset{D}{\overset{\|}{}}$$

$D = \mathbb{Q}_{p^2} e_\chi \oplus \mathbb{Q}_{p^2} e_{\chi^p}$ (I take $K = \mathbb{Q}_p$ here for simplicity)

$\begin{cases} \varphi(e_\chi) = e_{\chi^p} \\ \varphi(e_{\chi^p}) = -p^{-1} e_\chi \end{cases}$ $\begin{cases} g(e_\chi) = \chi(g) e_\chi \\ g(e_{\chi^p}) = \chi(g)^p e_{\chi^p} \end{cases}$ $g \in \mathrm{Gal}(F/\mathbb{Q}_{p^2})$, $\chi : \mathrm{Gal}(F/\mathbb{Q}_{p^2}) \to \mathbb{Q}_{p^2}^\times$ chara. $\chi^p \neq \chi$

$\begin{cases} g_\varphi(e_\chi) = e_\chi \\ g_\varphi(e_{\chi^p}) = e_{\chi^p} \end{cases}$ $g_\varphi \in \mathrm{Gal}(F/\mathbb{Q}_p)$, g_φ fixes $\sqrt[p^2-1]{-p}$ and is non trivial on $\mathbb{Q}_{p^2}$

$$\mathrm{Fil}^i(D \otimes_{\mathbb{Q}_{p^2}} F) = \begin{cases} \text{all if } i \leq -1 \\ F(\varpi_\chi a e_\chi + b e_{\chi^p}) \text{ if } i = 0 \\ 0 \quad \text{if } i \geq 1 \end{cases}$$

here $\varpi_\chi \in K$ is the smallest power of $\sqrt[p^2-1]{-p}$ so that $g(\varpi_\chi) = \chi(g)^{p-1} \varpi_\chi$, $g \in \mathrm{Gal}(F/\mathbb{Q}_{p^2})$

$(a, b) \in \mathbb{P}^1(\mathbb{Q}_p)$

$\rho(V) = \mathrm{triv}$, $\pi(V) = \left(\text{c-}\mathrm{Ind}_{\mathrm{GL}_2(\mathbb{Z}_p)\mathbb{Q}_p^\times}^{\mathrm{GL}_2(\mathbb{Q}_p)} \mathrm{DL}(\chi)\right)$, here $\mathrm{DL}(\chi)$ is the super-cuspidal representation of $\mathrm{GL}_2(\mathbb{F}_p)$ associated to χ that we view as a represent$^{\circ}$ of $\mathrm{GL}_2(\mathbb{Z}_p)\mathbb{Q}_p^\times$ via $p \mapsto 1$ and $\mathrm{GL}_2(\mathbb{Z}_p) \twoheadrightarrow \mathrm{GL}_2(\mathbb{F}_p)$. One can actually realize $\mathrm{DL}(\chi)$ over $\mathbb{Q}_p$, it has dim. $p-1$ and $\mathrm{DL}(\chi) \simeq \mathrm{DL}(\chi')$ iff $\chi' = \chi^p$ or $\chi' = \chi$.

<u>First covering of Σ_0</u>:

(some variant of) Σ_0 is a moduli space for formal groups of a special kind (Drinfeld). Using this interpretation, one can define a tower of coverings $\begin{matrix}(\Sigma_n)\\ \downarrow \\ \Sigma_0\end{matrix}$.

In the sequel, we will use only $\begin{matrix}\Sigma_1 \\ \downarrow \\ \Sigma_0\end{matrix}$. $\Sigma_1^{(0)} / \begin{pmatrix} p & 0 \\ 0 & p\end{pmatrix}^{\mathbb{Z}} \simeq \Sigma_1^{(0)}/_{\mathbb{Q}_{p^2}} \amalg \Sigma_1^{(1)}/_{\mathbb{Q}_{p^2}}$ new Frob: $\mathrm{Fr} = \varpi_D \circ \alpha$ ↳ p.Z "descent data"

Σ_1 is a quasi-Stein rigid space $/\mathbb{Q}_p$ endowed with an action of $\mathrm{GL}_2(\mathbb{Q}_p)$ and of $\mathcal{O}_D^\times$ such that $g \circ d = d \circ g$ if $\mathrm{val}(\det(g))$ even
↳ $D^\times$ here = qu. alg. $/\mathbb{Q}_p$! $g \circ d = \bar{d} \circ g$ if $\mathrm{val}(\det(g))$ odd.

$\mathrm{Fr} \circlearrowright \Sigma_1^{(0)}$ (s. linear) ↓ descends to Σ_1

The action of $\mathcal{O}_D^\times$ factors through $\left(\mathcal{O}_D/(\varpi_D)\right)^\times \simeq \mathbb{F}_{p^2}^\times$ and the Galois group of the covering $\begin{matrix}\Sigma_1\\ \downarrow\\ \Sigma_0\end{matrix}$ is precisely $\mathbb{F}_{p^2}^\times$.

Rq: In fact, there exists $u \in \mathcal{O}(\Sigma_0)^\times$ such that one has:

$$\Sigma_1 \simeq \Sigma_0\left(\sqrt[p^2-1]{u}\right) \simeq \left\{ (z,w) \in \Sigma_0 \times \mathbb{A}^{1,\mathrm{rig}} \;\middle|\; w^{p^2-1} = u(z) \right\}.$$

Don't know any formula for u.

Teitelbaum: constructs a semi-stable formal model of Σ_1 over $\mathcal{O}_F$.

<u>Theorem</u>: (isom. of $F_0 = \mathbb{Q}_{p^2}$-v.s.)

$$H^1_{HK}\left(\Sigma_1 \times_{\mathbb{Q}_p} F\right)^{(\chi^{-1}+\chi^{p})} \xrightarrow{\sim} \left(\mathrm{Ind}_{GL_2(\mathbb{Z}_p)\mathbb{Q}_p^\times}^{GL_2(\mathbb{Q}_p)} DL(\chi)^\vee\right) \otimes_{\mathbb{Q}_p} D^\vee$$

the map being compatible with $GL_2(\mathbb{Q}_p)$, φ and $\mathrm{Gal}(F/\mathbb{Q}_p)$.

Here, H^1_{HK} is Hyodo-Kato cohomology $\left(= H^1_{\log\text{-}\mathrm{cris}}\left(\text{special fiber} / \underset{1\mapsto 0}{W(\mathbb{F}_{p^2})}\right) \otimes_{\mathbb{Z}_{p^2}} F\right)$, extended to that situation by Grosse-Kloenne and $\chi^{-1}+\chi^{p}$ = sum of χ^{-1} and χ^{p} eigenspaces for the action of $\left(\mathcal{O}_D/(\varpi_D)\right)^\times \simeq \mathbb{F}_{p^2}^\times$.

To prove the theorem, use the fact that the special fiber of Teitelbaum's formal model is a tree of Deligne-Lusztig curves $x^{p^2-1} = \underset{\text{"normalized"}}{(y^p z - y z^p)^{p-1}}$ and that $H^1_{\mathrm{cris}}\left(\mathrm{DL\,curve}/\mathbb{Q}_{p^2}\right)^{(\chi^{-1}+\chi^{p})} \simeq DL(\chi)^\vee \otimes_{\mathbb{Q}_p} D^\vee$ (compatibly with φ).

Then, by a Mayer-Vietoris sequence, one has $H^1_{HK}\left(\Sigma_1 \times_{\mathbb{Q}_p} F\right)^{(\chi^{-1}+\chi^{p})} \simeq \bigoplus_{\text{vertice}} H^1_{\mathrm{cris}}\left(DL/\mathbb{Q}_{p^2}\right)^{(\chi^{-1}+\chi^{p})}$ (as $\chi \neq \chi^p$, the contribution of the intersection points is 0). $\square$

We thus have:

$$\underbrace{\Omega^1(\Sigma_1)^{(\chi^{-1}+\chi^{p})}}_{\left(= \Omega^1(\Sigma_1) \cap \Omega^1(\Sigma_1 \times_{\mathbb{Q}_p} F)^{(\chi^{-1}+\chi^{p})} = \left(\Omega^1(\Sigma_1 \times_{\mathbb{Q}_p} F)^{(\chi^{-1}+\chi^{p})}\right)^{\mathrm{Gal}(F/\mathbb{Q}_p)}\right)} \hookrightarrow \Omega^1(\Sigma_1 \times_{\mathbb{Q}_p} F)^{(\chi^{-1}+\chi^{p})} \twoheadrightarrow H^1_{dR}(\Sigma_1 \times_{\mathbb{Q}_p} F)^{(\chi^{-1}+\chi^{p})} \xrightarrow{\sim} \left(\mathrm{Ind}\, DL(\chi)^\vee\right) \otimes_{\mathbb{Q}_p} D_F^\vee$$

For $(a,b) \in \mathbb{P}^1(\mathbb{Q}_p)$, associate $\mathrm{Fil}^1 D_F^\vee := F(\varpi_{\chi^{-1}} a\, e_{\chi^{-1}} + b\, e_{\chi^{-p}})$

and define:
$$\Pi_{\chi,(a,b)} := s^{-1}\left(\mathrm{Ind}\, DL(\chi)^\vee \otimes_{\mathbb{Q}_p} \mathrm{Fil}^1 D_F^\vee\right) \hookrightarrow \Omega^1(\Sigma_1)^{(\chi^{-1}+\chi^{-p})}$$

<u>Proposition:</u> (i) $\Pi^\vee_{\chi,(a,b)}$ is a locally analytic representation of $GL_2(\mathbb{Q}_p)/\mathbb{Q}_p$ that is an extension:
$$0 \to \text{c-Ind}\, DL(\chi) \longrightarrow \Pi^\vee_{\chi,(a,b)} \longrightarrow \left(\mathcal{O}(\Sigma_1)^\vee\right)^{(\chi+\chi^p)} \to 0 .$$
(ii) In $\mathrm{Ext}^1_{\mathrm{loc\,an}}\left(\mathcal{O}(\Sigma_1)^{\vee(\chi+\chi^p)}, \text{c-Ind}\, DL(\chi)\right)$, one has:
$$[\Pi^\vee_{\chi,(a,b)}] = a[\Pi^\vee_{\chi,(1,0)}] + b[\Pi^\vee_{\chi,(0,1)}] .$$
(iii) $\Pi^\vee_{\chi,(a,b)}$ admits a universal unitary completion which is also a completion of $\text{c-Ind}\, DL(\chi)$.

<u>proof</u> (of (iii)): To prove that $\Pi^\vee_{\chi,(a,b)}$ admits a universal unitary completion, one uses, as for Σ_0, that one has:
$$\Omega^1(\Sigma_1) = \varprojlim_U \Omega^1(U) \qquad U = \text{affinoid}$$

Take U such that $(gU)_g$ covers Σ_1, and $q : \Omega^1(U) \to \mathbb{Q}_p$ a norm, then $q : \Pi_{\chi,(a,b)} \hookrightarrow \Omega^1(\Sigma_1)^{(\chi^{-1}+\chi^{p})} \hookrightarrow \Omega^1(\Sigma_1) \to \Omega^1(U) \to \mathbb{Q}_p$ is such that $(gq)_g$ gives the Fréchet topology on $\Omega^1(\Sigma_1)$, hence on any closed subspace. Now, I claim that the image of $\text{c-Ind}\, DL(\chi)$ in this universal unitary completion is dense. Otherwise one would get a non-zero map from $(\mathcal{O}(\Sigma_1)^\vee)^{(\chi+\chi^p)}$ to a GL_2-unitary Banach space*, which is impossible because the unitary completion of $(\mathcal{O}(\Sigma_1)^\vee)^{(\chi+\chi^p)}$ is zero. This follows from the fact

(* this Banach space being $\widehat{\Pi}^\vee_{\chi,(a,b)} / \overline{\text{c-Ind}\, DL(\chi)}$ assumed to be $\neq 0$)

that the dual of this unitary completion sits inside $O(\hat{\Sigma}_{1,F}) \underset{\mathbb{Q}_p}{\otimes} F^{(\chi+\chi^{-1})}$ ⑧

$\downarrow$ Teit.'s formal scheme over $\mathbb{Z}_F$

but $\dfrac{O(\hat{\Sigma}_{1,F})^{\chi^{-1}+\chi^{-p}}}{p\, O(\hat{\Sigma}_{1,F})^{\chi^{-1}+\chi^{-p}}} \hookrightarrow O(\hat{\Sigma}_{1,F} \times \mathbb{F}_{p^2})^{\chi^{-1}+\chi^{-p}} \hookrightarrow \prod_{\text{DL curve}} O(DL)^{\chi^{-1}+\chi^{-p}} = 0$.

as $\chi^p \neq \chi$ and $O(DL) \simeq (\mathbb{F}_{p^2})^{p-1}$. □

I would like to finish with this open question: the universal unitary completion of c-Ind DL(X) is very easy, it consists in functions

$$f: GL_2(\mathbb{Q}_p) \longrightarrow DL(X) \quad \text{such that} \quad f(k g) = k \cdot f(g) \quad (k \in KZ)$$

and such that $f(g)$ tends p-adically to 0 (inside the f.d. $\mathbb{Q}_p$-vector space DL(X)) when g moves away from KZ in the tree.

There should exists a subspace of explicit functions in that completion, depending on (a,b) and such that:

$$\widehat{\Pi^{\vee}_{X,(a,b)}} \simeq \frac{(\text{c-Ind } DL(X))^{\wedge}}{(\text{closure of that subspace})} = B(V)$$

What is this subspace?