

known  
w/ Genesier

$R_{k_0} = T_{k_0} \subset \mathbb{V}_B / \mathfrak{p}_{k_0} \mathbb{V}_B \quad \begin{pmatrix} a & b \\ v a^t & v b^t \end{pmatrix}$

then specialize in we (2.2)

$P_{0,0,1,1} = P_g$  - overconvergent

Conjecture If  $A/\mathbb{Q}$  abelian surface, irreducible,  
 then  $\exists g \in S_{2,2}(\Gamma)$ , eigen, such that  $P_{A,p}^{\vee} = P_{g,p}$ .  
 (generalization of Tamagawa-Shimura)

Thm: If  $A/\mathbb{Q}$  abelian surface

$\forall l \neq p$ ,  $A$  has purely toric red. mod  
 or good reduction.

$p$ : potential good ordinary

$P_{A,p}^{\vee} \cong P_{\tau, k_0}(\beta)$

$\Rightarrow \exists g \in S_{2,2}^+(\Gamma)$ ,  
 $P_{A,p}^{\vee} = P_{g,p}$

$GL_2$

$$\Gamma(W) \subset \Gamma \subset SL_2(\mathbb{Z})$$

torsion free:

$$E = \Gamma \backslash \mathbb{H}_1 \times \mathbb{C} / \mathbb{Z} \oplus \mathbb{Z}$$

$\downarrow$

$$Y_\Gamma = \Gamma \backslash \mathbb{H}_1$$

$$\gamma(z, v) = (\gamma(z), j(\gamma, z) v)$$

$$(z, v)(a, b) = (z, v + za + b)$$

$$L_{\mathbb{C}}(E/Y) = \Gamma \backslash \mathbb{H}_1 \times \mathbb{C}$$

$$\underline{\omega} = \Omega^1_{E/Y} = \Gamma \backslash \mathbb{H}_1 \times \mathbb{C}$$

$$\text{w/ action: } \gamma(z, v) = (\gamma(z), j(\gamma, z) v)$$

$$k \in \mathbb{Z} \quad \frac{\omega^{\otimes k}}{\downarrow}$$

$$\Gamma \mapsto \mathcal{O}_{\mathbb{H}_1}^*$$

$$\gamma \mapsto (z \mapsto j(\gamma, z)^k)$$

$Sp_4$

$$\Gamma(W) \subset \Gamma \subset Sp_4(\mathbb{Z})$$

$$A = \Gamma \backslash \mathbb{H}_2 \times \mathbb{C}^2 / \mathbb{Z}^2 \oplus \mathbb{Z}^2$$

$\downarrow$

$$Y_\Gamma = \Gamma \backslash \mathbb{H}_2$$

$$\gamma(z, v) = (\gamma(z), j(\gamma, z) v)$$

$$(z, v)(a, b) = (z, v + za + b)$$

$$L_{\mathbb{C}}(A/Y) = \Gamma \backslash \mathbb{H}_2 \times \mathbb{C}^2$$

$$\underline{\omega} = \Omega^1_{A/Y} = \Gamma \backslash \mathbb{H}_2 \times \mathbb{C}^2$$

$$\gamma(z, v) = (\gamma(z), j(\gamma, z) v)$$

$$k = (k_1, k_2), k_1 \geq k_2 \quad \frac{\omega^k}{\downarrow} = \Gamma \backslash \mathbb{H}_2 \times W_k$$

$\downarrow$

$$\gamma \mapsto (z \mapsto \rho_k(j(\gamma, z)))$$

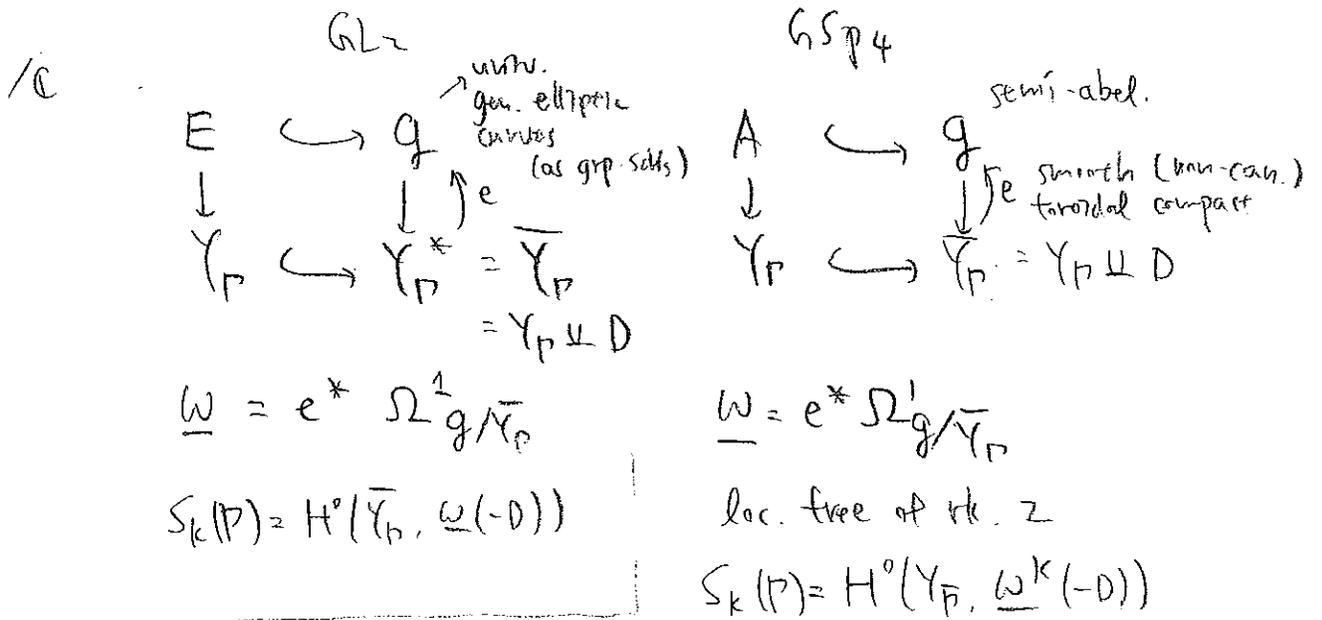
$$\underline{\omega} \longleftarrow j(\gamma, z), k \in \mathbb{Z}, 0$$

$$k = (k_1, k_2), W_k = (\Lambda^2 \mathbb{C}^2)^{\otimes k}$$

$\rho_k: GL_2 \rightarrow \text{Sym}^{k_1-k_2} \otimes \det^2(\mathbb{C}^2)$

$\rho$ -adically will make sense for any  $k$

$\therefore \underline{\omega}$  is trivial on the mod. locus

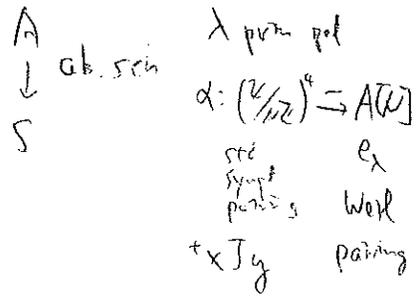
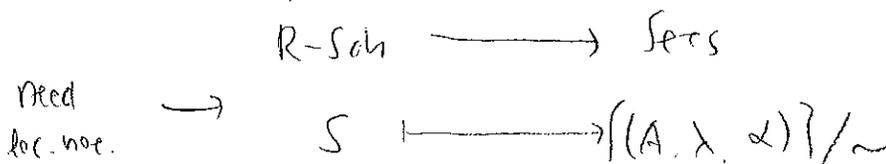


Over  $\mathbb{Z}[\frac{1}{N}, \frac{1}{6}] = \mathbb{R}$

no longer true  $Sp_4$

1) Mumford (GIT)

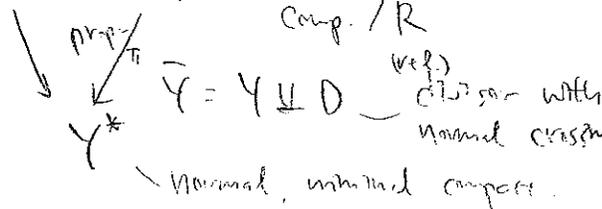
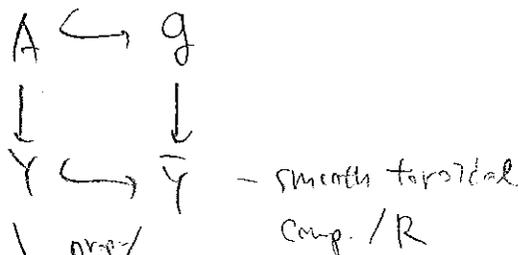
$N \geq 3$



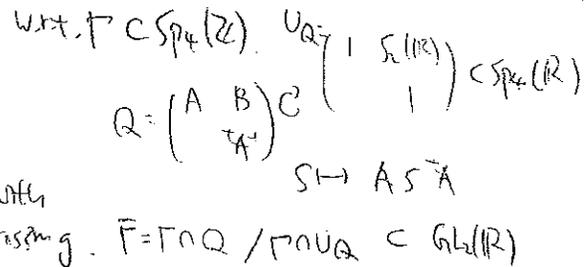
is representable by  $Y/R$   
quasi-proj, geom. connected  
smoothly with univ. ab.



2) Faltings-Chac — semi-abel. sch.



$\bar{Y}$  needs the choice of an admis. rational polyhedral cone decomp. of the cone  $S_2(\mathbb{R}) \subset S_2(\mathbb{R})$



NumArd:

$$\mathcal{T} = \underline{\text{Isom}}(D_Y, \underline{\omega})^{\mathfrak{g}_{GL_2}} \quad (\varphi, g) \mapsto \varphi \circ g$$

$$\downarrow \mathfrak{g}_{GL_2}$$

$$Y_{P/R}$$