Local-global compatibility in the p-adic Langlands correspondence for GL_2 over $\mathbb{Q}$

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1. Introduction

The local-global compatibility conjecture is a statement about the multiplicity with which a given odd irreducible continuous two-dimensional representation ρ of $G_{\mathbb{Q}}$ appears in the p-adically completed cohomology of modular curves.

More precisely, this multiplicity space is a p-adic Banach space equipped with a continuous action of $\mathrm{GL}_2(\mathbb{A}_f)$, and the conjecture predicts its precise structure.

A reference for the conjecture is the preprint "A local-global compatibility conjecture in the p-adic Langlands programme for $\mathrm{GL}_{2/\mathbb{Q}}$", which is available on my web-page.

In my lectures I will

(1) State the conjecture, both in the form given in the above preprint, and in a stronger form that incorporates deformations of Galois representations.
(2) Explain some consequences of the conjecture, including:
    (a) cases of the Fontaine-Mazur conjecture;
    (b) a conjecture of Kisin characterizing the Galois representations attached to finite slope overcongvergent eigenforms in terms of their local structure at p (these first two consequences are also both explained in the above preprint);
    (c) the construction of multi-variable p-adic L-functions attached to families of modular forms;
    (d) a mod p local-global compatibility statement, which gives a representation theoretic reformulation and strenghtehning both of Serre's conjecture and of classical mod p multiplicity one statements for the cohomology of modular curves.
(3) Explain a proof of the conjecture (in its strong deformation theoretic form, under some hypotheses on the structure of $\overline{\rho}$), which is contingent on a purely local statement (a deformation theoretic formulation of the local p-adic Langlands correspondence) which is the subject of ongoing work of Colmez.

A sketch of the work of Colmez that is relevant to item (4) can be found in the introduction of the preprint "The Fontaine-Mazur conjecture for GL_2" available on Mark Kisin's web-page.

Background on the p-adically completed cohomology of modular curves, and its relation to the eigencurve and to p-adic L-functions, may be found in my papers "On the interpolation of systems of eigenvalues attached to automorphic Hecke eigenforms", in Inventiones (particularly §4), and "p-adic L-functions and unitary completions of representations of p-adic groups", in Duke (particularly §§4,5). Both these papers are also available on my web-page.

References

[1] M.Emerton, *A local-global compatibility conjecture in the p-adic Langlands programme for* $\mathrm{GL}_2/\mathbb{Q}$, preprint (2005).
[2] M.Emerton, *p-adic L-functions and unitary completions of p-adic reductive groups*, Duke Math.J. **130** (2005).
[3] M.Emerton, *On the interpolation of systems of eigenvalues attached to automorphic Hecke eigenforms*, Invent.Math. 164 (2006).
[4] M.Kisin, *The Fontaine-Mazur conjecture for* GL_2, preprint (2006).