Courses in Mathematics

(2015-2016)

This document gives a brief description of the various courses in calculus and some of the intermediate level courses in mathematics. It provides advice and pointers for planning your course selections. If you are a Mathematics Concentrator, or are considering entering the Mathematics Concentration, and if you are seeking some overview of the courses and how they fit together, then this document is for you. However, the guidelines presented below are exactly that: guidelines. Keep them in mind when you are deciding how to structure your program, but be sure to talk to your advisor in the Mathematics Department or to the Director of Undergraduate Studies before you turn in your study card each semester.

1 Calculus

Math 1a/b is the standard first-year calculus sequence. If you are thinking about majoring in math and have not taken calculus before, take Math 1 as soon as possible! If you have had a year of calculus in high school, and if you have passed the Advanced Placement examination in BC Calculus with a score of 4 or better, then you may be advised to begin with Math 21 a/b, the second-year calculus sequence.

If you scored a 5 on the BC Calculus exam and if you are advised to take Math 21 a/b, then you may wish to consider taking Math 23 or Math 25 or 55 instead of Math 21. Be warned: Math 23, 25 and 55 are intense but very rewarding courses, and both 25 and 55 require extensive work outside the classroom. To succeed in the latter two, you must be very committed to mathematics from the start.

Regardless of which calculus course you take, keep in mind that it is important to absorb ideas thoroughly. It's a bad idea to push yourself too far too fast.

For more guidance on choosing your first math course at Harvard please read the pamphlet "Beyond Math 1: Which math course is for you?", which you can obtain from Cindy Jimenez, the Undergraduate Program Coordinator (room 334), or from the undergraduate section of the Department's web site.

2 How to structure a good program

No single program is ideal for all math concentrators. You should design your curriculum based on your background, interests, and future plans. You are strongly urged to consult with your academic advisor or with the Director of Undergraduate Studies in deciding which courses are best suited for you. Do not plan to meet with your advisor on the

day study cards are due, since advisors usually don't have more than a few minutes to spend with each student that day. Make an appointment with your advisor well before study cards are due. You should allot about half an hour, so you can discuss your plan of study in depth.

Learning to write proofs

Math 23, 25, 55, 101, 112, and 121 are six courses in which you learn to write proofs, meeting (often for the first time) a style of mathematics in which definitions and proofs become part of the language. Students are generally advised not to take any upper-level math courses before completing (or, at least, taking concurrently) one of these.

- Math 101 serves three main goals. It lets a student sample the three major areas of mathematics: analysis, algebra, and topology/geometry; it introduces the notions of rigor and proof; and it lets the student have some fun doing mathematics. If you are considering concentrating in Mathematics but are not sure that you are ready to take Math 23, 25 or 55, or if you simply want a glimpse of what "higher" math is all about, you are urged to include Math 101 early in your curriculum. Math 101 can be taken concurrently with either Math 21a or 21b. This course is only offered in the fall. If you have had some experience with rigorous proofs and want a different taste of "higher" math, you might consider Math 152 in the fall. Neither Math 101 nor Math 152 is appropriate for people from Math 25, Math 55 or (with rare exceptions) Math 23.

- Math 23, 25 and 55 are the three introductory courses for students with strong math interests. They are geared towards new students. Math 25 and 55 are much more intensive than Math 23, but require much more out of class time. Students who don't wish to make the time commitment will do well to choose Math 23. Meanwhile Math 55 should be taken only by students with *extensive college level math backgrounds*. Each year several first-year students ask to skip the Math 25/55 level and start with Math 122 or another 100-level course. The Department, based on many years of experience, *strongly discourages* this. Even if you have taken several years of math at another university, even if you have seen every topic to be covered in Math 25 or 55, you will not be bored in these accelerated courses. The topics covered in Math 25 and 55 are not as important as the level and the depth of mathematical maturity at which they are taught. Taking Math 25 or 55 is the most intense mathematical experience you are going to have in any Harvard course, shared with the most talented of your peers. You may learn more advanced material in other 100- and 200-level courses, but never with the same speed and depth as in Math 25 or 55. These courses are not taught in any other university because no other university has the same caliber of first-year mathematicians. And

the courses are simply a lot of fun. Many students who have skipped 25 and 55 have been dissatisfied with their decision. In any event, you *must* speak with the Director of Undergraduate Studies if you plan to skip the Math 21-55 level.

- Math 112 and Math 121 are courses suitable for students from Math 21, and they provide an alternative entry-point for the department's more advanced courses in Analysis and Algebra respectively. They should not be normally be taken by students who have been through Math 23 or 25. If you are a sophomore and have taken Math 21 but are not yet comfortable with writing proofs, then consider including these courses in your plan of study.

Key courses at the 100 level

If you have taken Math 23, 25 or 55, or if you have taken Math 21 and gained some experience in writing proofs through courses such as Math 101, 112 and 121, then you are ready to take some of the courses at the 100-level that form the core of the Mathematics curriculum. Most of the courses at this level can be classified as belonging to one of the three main streams of mathematics: "Analysis", "Algebra" and "Geometry and Topology". Courses belonging to these areas are numbered in the ranges 110–119, 120–129 and 130–139 respectively. In each stream, there are two courses which are regarded as "core" courses, making a total of six central courses. These are:

- Math 113. Analysis I: Complex Function Theory
  Math 114. Analysis II: Measure, Integration and Banach Spaces
- Math 122. Algebra I: Theory of Groups and Vector Spaces
  Math 123. Algebra II: Theory of Rings and Fields
- Math 131. Topology I: Topological Spaces and the Fundamental Group
  Math 132. Topology II: Smooth manifolds

It is not necessary to include all six of these courses in your plan of study, but here are some points to bear in mind.

- Students from Math 55 will have covered in 55 the material of Math 122 and Math 113. If you have taken 55, you should look first at Math 114, Math 123 and the Math 131-132 sequence.
- With the exception just noted, you should consider including Math 122 early on in your curriculum. Algebra is a basic language of modern mathematics, and it is hard to comprehend advanced material without some familiarity with groups and related topics in algebra. The same remark applies to Math 123, to a lesser degree.

By the same token, Math 113 should also be taken early on as Complex Analysis is used in many other fields of mathematics. You will also find the topology you learn in Math 131 useful in many other areas: amongst other things, it provides the mathematical language with which to discuss continuity and limits in wide generality.

- Math 123 cannot be taken before Math 122; but in the other two streams, the courses can be taken in either order. Thus, Math 114 can be taken before or after Math 113, and the same applies to Math 131 and 132.

- You should try to fulfill the *distribution requirement* (i.e., the requirement to take at least one course in analysis, algebra, and geometry) early in your academic career. By your junior or senior year, you should be exposed to the main branches of mathematics; then you can choose the department's advanced courses. In any case, most 200-level courses assume (at least informally) familiarity with the basic tools of analysis, algebra, and topology.

Other courses at the 100 level

At this level, there are many other courses to choose from: Number theory in Math 124 or Math 129, Differential Geometry in Math 136, Probability in Math 154, Logic and Set Theory in Math 141 and Math 143, amongst others.

- It is a good idea to take a tutorial (Math 99r) during the sophomore or junior year. Many students found the tutorial to be one of the best courses they took at Harvard. Tutorials generally satisfy the Math Expository requirement and often lead to senior thesis topics. More about tutorials appears below.

- Students wishing to take a rigorous course in mathematical logic in years when Math 141, 142, 143, or 144 are not offered at Harvard should consider taking logic courses at M.I.T. In any event, the Harvard courses offer a good introduction to model theory, set theory and recursion theory — the three main branches of Mathematical Logic. Students interested in the more philosophical aspects of logic and/or in proof or set theory may want to take Philosophy 143, and those interested in mathematics of computation should look into Computer Science 121 and some of the other theoretical CS courses.

- Students interested in Combinatorics should look at Math 155, and may also want to look up M.I.T.'s listings in that area. If you want M.I.T. courses to count for the concentration credit, you must get permission in advance from the Director of Undergraduate Studies, Prof. Jacob Lurie (lurie@math).

- Students are encouraged to take courses from a variety of professors in the department and not just to "follow" one teacher. It is advisable to be exposed to different views and styles of doing mathematics.

200-level courses

100, 200 – What's the Difference?

The difference between 100-level and 200-level courses is fairly easy to summarize: 100-level courses are designed for *undergraduates,* whereas the 200-level courses are generally designed for *graduate* students. As far as course material goes, the 100-level courses are designed to offer a comprehensive view of all the major fields in pure mathematics. They emphasize the classical examples and problems that started each field going and they all lead to one of the fundamental results that motivates the further development of the field. In contrast, a 200-level course will assume you understand the basic ideas of a field. A 200-level course will set out the systematic, abstract foundations for a field and develop tools needed to get to the present frontiers.

The 100-level courses give you a good overview of mathematics, they foster intellectual growth, and they prepare you for your chosen career. This is not true of 200-level courses. These courses assume that you are interested in the subject, and that you are already fairly certain of becoming an academic mathematician. The amount you learn in such a course is often also entirely up to you. Your prerequisites, though correct according to the course catalog, may be entirely inadequate. Many courses are paired into 100-level and 200-level sequences:

Corresponding 100-level, 200-level Courses

Math 114	-¿	Math 212a,b	(Real Analysis)
Math 113	-¿	Math 213a,b	(Complex Analysis)
Math 122/123	-¿	Math 221	(Algebra)
Math 129	-¿	Math 223a,b	(Algebraic Number Theory)
Math 131	-¿	Math 231a,b	(Algebraic Topology)
Math 132/136	-¿	Math 230a,b	(Differential Geometry)
Math 137	-¿	Math 232a,b	(Algebraic Geometry)

Other 200-level courses are harder to classify, but cover topics equally central to modern mathematics. For example, Math 222 is a course on Lie Groups and Lie Algebras that draws on background material from Analysis, Algebra and Geometry.

Skipping 100-level Precursors

Students are *strongly* discouraged from taking any 200-level course before taking its 100-level precursors. Although it is possible in principle to learn a general abstract topic on the basis of the logic of its definitions and theorems alone, it is almost impossible to appreciate their significance and "feel" without studying the more down-to-earth background which led to them. *Moreover,* students are well advised to take basic classes in algebra, topology, and analysis before exploring the graduate curriculum: often a basic familiarity with other areas will be an assumed prerequisite. Certainly, it can't hurt. However, even this *may* not suffice.

Some graduate courses (notably 212a, 221a, 231a) often conform better to undergraduate expectations (set material, careful pace, motivation); the best way to tell whether this is going to happen is to go to the class yourself and find out. Beware, though: often these courses start in a user-friendly way (presenting simple definitions, for example), then speed up tremendously as time goes on.

Why Take 200-level Courses?

The reasons for *not* taking 200-level courses are legion. However,there are some equally good reasons for taking them. You will be treated like a graduate student, which is good if you want to be treated like one. There isn't much review of topics you may have already covered, requirements are fairly minimal, and, most importantly, you can learn a lot of substantial mathematics. (If this is what you want, tutorials are another good option. While they are undergraduate courses, one generally learns graduate material in them.)

A student who is considering graduate school in mathematics may want to include at least one 200-level course in his or her program (and, likewise, write a senior thesis) to get a taste of the likes of graduate school.

3 Other types of course

Tutorials

Tutorials are not required, but many students take a tutorial during their sophomore or junior year. Typically two tutorials are offered every semester.

Tutorials (Math 99r) are generally directed by graduate students, and have four to eight students in them. They tend to be less formal and structured than regular courses, yet require more involvement on the part of the students – students have to make presentations and write papers. Very frequently a topic studied in a tutorial leads naturally to a senior thesis. And the paper written for the tutorial generally satisfies the Math expository requirement.

The department places a description of the fall tutorials into concentrators' registration envelopes in September; a description of the spring tutorials is e-mailed to the concentrators e-mail list in January. Descriptions also appear during the first week of that semester on the undergraduate bulletin boards (one opposite room 320, and one near room 503 in the Math Department). The descriptions also appear on the Math Department's website at http://www.math.harvard.edu/. Often, tutorials get previewed at Math Table meetings. A special organizational meeting for tutorials is held in the first week of the fall semester. The spring semester tutorials are organized in the first week of that semester; see the Undergraduate bulletin boards for announcements.

Ordinarily only one Math 99r can count towards the concentration requirements.

All questions regarding tutorials may be addressed to the Director of Undergraduate Studies or the Undergraduate Program Coordinator, Cindy Jimenez (cindy@math).

Reading Courses (60r and 91r)

Honors candidates in their Senior year can choose to enroll in Math 60r to allow more time for thesis work. You can take Math 60r in the fall and/or spring semester. Math 60r is SAT/UNS only and does not count for concentration requirements. A student taking Math 60r in the fall must submit a one-page plan of thesis (including at least a preliminary bibliography) to Cindy Jimenez (rm. 334) **by 4 pm** of the last day of the fall reading period in order to pass.

If you want to learn a particular topic not covered in a regular course or a tutorial, you may consider taking Math 91r. For this you must find a faculty member willing to supervise your reading, as well as secure approval from the Director of Undergraduate Studies. Make sure that you, your supervisor, and the Director of Undergraduate Studies clearly agree on the topic, structure, frequency of meetings, and the grade requirements before you sign up for 91r. You should know exactly what is expected of you and how much guidance to anticipate. Ordinarily, Math 91r will not count for concentration requirements.

Note that Math 60r, 91r, and 99r require the signature of the Director of Undergraduate Studies on your study card.

Cross-registration at M.I.T.

Students may cross-register to take a course at M.I.T. This may be a useful option in years when a particular course is not offered at Harvard. Logic and Combinatorics offerings at M.I.T. have proven especially popular with Harvard students. Generally, classes at M.I.T. start a week before Harvard's in the fall, and contemporaneously with Harvard's in the spring. You may get concentration credit for M.I.T. courses, but consult

the Director of Undergraduate Studies *before* registering. Cross-registration petitions can be obtained at the Registrar's office or from your House's Senior Tutor.

If you are taking an M.I.T. course, you don't have to walk all the way down Mass. Ave. or even pay for the bus to get to class: you can use the Harvard Medical Area (M2) shuttle bus, which runs from Quincy Square (in front of Lamont) straight to M.I.T.

Related fields

Keep in mind that the concentration requirements for Mathematics require twelve half-courses, but only eight of those need to be listed under "Mathematics" in the Course Catalog. You are encouraged to round out your studies by including courses listed as "Related Fields" in the mathematics section of the *Handbook for Students*.

4 Sample Programs

The programs listed below should not be followed literally – they may not be balanced in workload between the fall and the spring semesters, nor are all the courses listed necessarily offered every year. They are examples designed to demonstrate the range of possibilities. You should determine your own program in consultation with your math faculty advisor or the Director of Undergraduate Studies.

(a) If you start with Math 1 a/b in your first year, you can continue with Math 21 a/b as a sophomore. Students who start with Math 1b in the fall of their first year normally take Math 21a in the spring; some choose also to take Math 21b concurrently with 21a in order to get "in sync." Some students who start with Math 1 a/b sequence freshman year and do extremely well may choose to take Math 23 or 25 their sophomore year, instead of Math 21 (if you are considering doing this, you should talk to Jacob Lurie, the Director of Undergraduate Studies (room 514)). Otherwise, you'll get a first feel for proofs and abstraction by taking Math 101, 112 or 121. A possible schedule is:

FR	**SO**	**JR**	**SR**
Math 1a	Math 21a	Math 112	Math 113
Math 1b	Math 21b	Math 121	Math 131
CS 50	CS 51	Physics 15a	Math 122
	Math 101	Stat 110	Phil 144

(b) Students who start with 21 a/b in their first year can take 101 either concurrently with Math 21 or in their sophomore year along with with 112, 122 and 131. Students are also encouraged to take Physics 15 a/b/c or Computer Science 51 to broaden their understanding of how mathematics applies to other disciplines. Students who wish to

write a senior thesis often take reading course or a 200-level course in the field of their senior thesis during their senior year.

FR	**SO**	**JR**	**SR**
Math 21a	Math 122	Math 141	Math 231a
Physics 15a	Math 131	Math 124	Math 114
Math 21b	Comp Sci 51	Math 99r	Math 231b
Math 101	Math 112	Math 132	Ec 2052

(c) A student with a strong interest in mathematics, or a strong mathematical background would most likely start with Math 23, 25 or 55 during the first year. His or her sample program might look like this:

FR	**SO**	**JR**	**SR**
Math 23a or 25a	Math 122	Math 114	Math 60r
Physics 15a	Math 131	Math 99r	Math 212a
Math 23b or 25b	Math 123	Math 129	Math 222
Physics 15b	Math 113	Math 132	Math 137

(d) Consider a student with a strong interest in mathematical physics, concentrating in Mathematics as a primary field and Physics as the secondary one, and who started with a Math 23a,b or Math 25a,b sequence and the Physics 16, 15b, 15c sequence. Some of the 100-level math courses of particular interests to physicists are Math 115, 132 and 136. The sequel to Physics 15c is Physics 143a,b. Choosing some of the math and physics courses with the most conceptual interaction, you might come up with the following to fulfill the math portion of the requirements. (You should consult with the Physics Head Tutor to plan the physics portion.)

FR	**SO**	**JR**	**SR**
Math 23a or 25a	Math 131	Math 115	Math 230a
Physics 16	Math 122	Math 132 or 136	Math 99r
Math 23b or 25b	Phys 15c	Physics 143b	Math 230b
Physics 15b	Phys 143a	Phys 181	Math 123

(e) A primary/secondary honors major in Mathematics and Computer Science is common. If you choose this option, you are required to write a thesis which applies ideas of computer science to a topic in pure mathematics, or vice versa. Mathematics courses of particular value here would be Math 141 (introduction to mathematical logic), Math 142 (Recursion Theory), Math 124 (number theory including primality tests and applications to codes), Math 130 (on axiomatic foundations of geometry), and Applied Math 107 (combinatorics). A possible program when Math is the primary which fulfills the

Math requirements is given below. Consult with the Computer Science Head Tutor to plan that portion of your course work.

FR	**SO**	**JR**	**SR**
Math 21a	Math 121	Math 122	Math 141
Math 101	CS 121	CS 207 or 226r	Math 191
Math 21b	Math 112	AM 107	Math 142
CS 51	CS 124	Math 130	Math 124 or AM 111 or Math 152